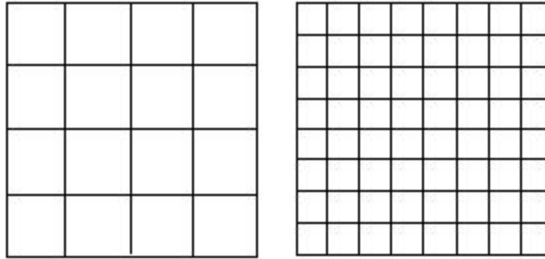


## Show Notes:

- [17:20] Fractal Dimension
  - [https://en.wikipedia.org/wiki/Hausdorff\\_dimension](https://en.wikipedia.org/wiki/Hausdorff_dimension)
  - [https://en.wikipedia.org/wiki/Fractal\\_dimension#Role\\_of\\_scaling](https://en.wikipedia.org/wiki/Fractal_dimension#Role_of_scaling)
  - [https://en.wikipedia.org/wiki/How\\_Long\\_Is\\_the\\_Coast\\_of\\_Britain%3F\\_Statistical\\_Self-Similarity\\_and\\_Fractional\\_Dimension](https://en.wikipedia.org/wiki/How_Long_Is_the_Coast_of_Britain%3F_Statistical_Self-Similarity_and_Fractional_Dimension)
  - <http://math.bu.edu/DYSYS/chaos-game/node6.html>: Fractal dimension is a measure of how "complicated" a self-similar figure is. In a rough sense, it measures "how many points" lie in a given set.
  - A good example: a line can be broken into  $N$  different lines that each need to be scaled by a factor of  $N$  to get the original line:  
---- (This is a line of length 4 = 4 lines of length 1)

Similarly, a square can be broken into  $N^2$  different copies that each, when scaled by  $N$  yields the original square:



A square cut into 25 identical copies, each when scaled by a factor of 4 produces the original square. Finally, a cube can be broken to  $N^3$  different identical cubes, each when scaled by a factor of  $N$  yields a cube of the original volume. But what about a weird curve like this?



We can approach this from a different angle: Scale the Sierpinski triangle by 2, and you get 3 different copies of it. What does this mean though? Is it 1 dimensional? It's just a bunch of lines after all! But it's also a triangle...?

The fractal dimension answers the question for a given curve:

“What is the exponent  $D$  I need to raise an integer  $N$  by to produce  $N^D$  identical curves that each, when scaled by  $N$  give me the original curve?”

Formally, this is represented as:

$D = -\log(S)/\log(N)$  where  $S$  is the total number of identical copies (this notation is not canon!), and  $N$  is our scaling factor.

Going back to our problem with the freaky triangle above,

$$N = 2, N^D = 3 \Rightarrow D = \log(3)/\log(2) \approx 1.585$$

We can see it's somewhat 1-D and somewhat 2-D, but to a very precise degree. That's the power of fractal dimension, it tells us exactly what the dimension of a curve is to a degree that matches our intuition!

- More fractals:

[https://en.wikipedia.org/wiki/List\\_of\\_fractals\\_by\\_Hausdorff\\_dimension](https://en.wikipedia.org/wiki/List_of_fractals_by_Hausdorff_dimension)

- [23:55] Dirac Notation and Hilbert Space

- [https://en.wikipedia.org/wiki/Bra%E2%80%93ket\\_notation](https://en.wikipedia.org/wiki/Bra%E2%80%93ket_notation)
- [26:55] 10 Martini Problem
  - The name was coined by Barry Simon in this 1982 [article](#) (page 487):
  - **The Ten Martini Problem:** *The almost Mathieu operator has a Cantor spectrum.*
  - The name comes from the fact that Mark Kac\* has offered ten martinis to anyone who solves it. [...] Actually, Kac said "has all its gaps there", so perhaps one should solve instead:
  - **The Ten Martini Problem:** (Strong Form, or should it be Dry Form)...
  - [\*] Marc Kac, public communication at 1981 AMS Annual Meeting.
  - <https://annals.math.princeton.edu/wp-content/uploads/annals-v170-n1-p08-p.pdf>
- [28:10] Mathieu Operator
  - [https://en.wikipedia.org/wiki/Almost\\_Mathieu\\_operator](https://en.wikipedia.org/wiki/Almost_Mathieu_operator)
- [34:10] Julia Sets and Mandelbrot Set
  - <http://www.alunw.freeuk.com/mandelbrotroom.html>
  - [https://en.wikipedia.org/wiki/Mandelbrot\\_set](https://en.wikipedia.org/wiki/Mandelbrot_set)
- [51:10] Blind Spot
  - <https://www.exploratorium.edu/snacks/blind-spot>